Proof-Writing, Peer Review, and Portfolios: Getting Your Math Department to Go WAC

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Welcome Remarks

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Writing Requirements and the Mathematics Department

A Brief History
Teach the writing process – planning, drafting, revising, and editing;
Assign genres authentic to the discipline, like analytical essays, lab reports, case studies, term papers, among others;
Require students to submit 4000 words (or fifteen pages) of polished writing;
Emphasize the central role of writing in the course by allocating a substantial portion of the final grade to writing assignments.
Spring 2003 – The Math/CS Department submitted a request to designate Discrete Mathematics (Math 211) and Linear Algebra (Math 262) as a WR course.

Rationale:

- The writing skills of the upper-level math students were not as strong as desired.
- A WR course in the major would create a more effective gateway to the major by focusing on proof-writing earlier in the students’ mathematical careers.
Theorem: The sum of any two odd integers is even.

What tools would you need and what steps would you take to create a rigorous mathematical argument to prove this theorem?
1. Understanding the logical form of the theorem.
2. Understanding the content of the theorem.
3. Mathematical Analysis.
4. Decide what proof technique to use.
5. Writing the proof.
6. Revision, revision, revision.
“In both of these courses there will be many small writing assignments. Several times per semester (at least once every two weeks) these writing assignments will be edited and revised. It is important for a student to learn from their mistakes and to learn to polish their writing.” - Spring 2003
___ Students engage in planning exercises.
___ Students write a proposal or prospectus.
___ Students revise one paper.[multiple but not all]
___ Students revise all papers.
___ Students perform in-class peer review.
___ Students discuss drafts with instructor.
___ Students perform peer review outside of class.
___ Students revise after receiving written feedback from the instructor.
___ Students revise after receiving individualized oral feedback from the instructor.
Spring 2010 – The Math/CS Department submitted a request to designate Discrete Mathematics (Math 211) as the department WR course.

Rationale for change:

- “Proof writing is taught through repetition instead of revision (daily homework, typically including 2-3 proofs), since pedagogically revision does not work. For each homework assignment, students will receive feedback on the mathematics and on their writing. New homework problems will be assigned that address these same issues. “

Course Components: Homework and Exams
___ Students engage in planning exercises.
___ Students write a proposal or prospectus.
___ Students revise one paper.
___ Students revise all papers.
___ Students perform in-class peer review.
___ Students discuss drafts with instructor.
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___ Students revise after receiving individualized oral feedback from the instructor.
An introduction to fundamental mathematical concepts used in mathematics as well as computer science, with an emphasis on writing mathematical arguments. The course presents the principles of mathematical logic and methods of proof such as direct and indirect proofs and mathematical induction. Other topics include sets, functions, relations, matrix algebra, and techniques from elementary combinatorics and graph theory.

Prerequisite: 170 or COMP 131 or departmental placement. This course fulfills the WR graduation requirement. This course fulfills the QR graduation requirement.
This is it...the course where we invite students to learn what math is “really all about.”

Students will be introduced to some new mathematical content, but the bulk of the time and energy in this course is spent in teaching the science and art of “the proof.”

Even students who self-identify as being excellent in mathematics will often face difficulty with the adjustment to this way of thinking and communicating.
Balancing Content and Process

Getting Our Department to “Go WAC”
Spring 2011, Sarah (1\textsuperscript{st} time teaching this course)

- After attending the WR orientation workshop, Sarah was unsure about how the WR recommendations were incorporated into the course and had discussions with Noreen about restructuring the course to achieve a balance between content and the writing process.

- Changes:
  - Journals (prompts)
  - Portfolio (peer-review, \LaTeX)
  - Groupwork
Fall 2012, Jennifer (2\textsuperscript{nd} time teaching the course)

- Adopted a course structure very similar to Sarah’s.
- Changes:
  - New and updated journal prompts
  - Portfolio details (assignment sheet)
  - Additional groupwork
  - \textbf{Note:} Jennifer reduced the number of portfolio problems from 15 to 10. There are trade-offs here, and the perfect composition of problems is yet to be found.
Spring 2012, Sarah (2nd time teaching the course)

- Kept the class structure largely the same.

- Changes:
  - Writing associate
  - Additional journals (weekly + 5 prompts)
  - Cover letter for portfolio (new to me, not Jen)
Current Course Components:

- **Groupwork**: nongraded
- **Journal**: 5%; *low-stakes* (graded for completion)
- **Homework**: 15%; *collected every meeting* (MR)
- **Quizzes**: 5%; *three total, staggered between exams*
- **Portfolio**: 15%; *peer-reviewed; written in LaTeX*
- **Exams**: 60%; *two midterms and a final*
Typical Day (75-minute sessions, twice a week)

- Immediately (before class/start of class) work in peer-review groups.
- Lecture/lesson.
- End class with groupwork (5-20 minutes).

**Outside of class:** reading textbook before/after class, reviewing comments on returned homework, incorporating feedback from peer review into portfolio problems, reading and understanding groupwork solutions, finishing homework, finishing 1-3 portfolio problems for peer review, and journaling.
Elaboration on Course Components
Groupwork is ungraded; problems related to concepts from the day’s lecture are written on prepared handouts and students work in small groups or pairs to solve them.

Groupwork problems are similar to homework/portfolio problems.

Solutions to groupwork problems are posted before the homework/portfolio problems are due.

Students often remark in evaluations that they appreciated the groupwork.
Groupwork is a fairly common component of mathematics classes.

In this course, we thought of groupwork as a time to undertake the first few stages of mathematical writing:

1. Understanding definitions and the logical forms of related problems and theorems.
2. Mathematical Analysis.
3. Decide what proof techniques to use.
**Journal Details:** Students must write one entry a week, summarizing a learning moment from the week. In addition, they will respond to particular prompts, provided by the instructor.

- Sarah provided handouts on “how to journal” and about 5 prompts throughout the semester (topics given on an upcoming slide).
The main goal of journals: transfer of knowledge.

- We wanted the students to analyze the patterns in the instructor and peer feedback so that they mindfully incorporated improvements into their work.
- Also, we focused their attention on peer-review and the writing process.

Flow Chart:

Groupwork → Homework/Portfolio → Peer-Review & Returned Homework with Comments → Revised Portfolio
1. Your Auto-Math-ographical Tale
2. Discrete Math: This isn’t Calculus anymore
3. Peer Review Groups
4. Patterns
5. Summary of Your Writing Journey and Advice for New Math Writers
Guiding Questions:

- What is your history with mathematics? What experiences, if any, in your education encouraged you to continue studying mathematics? Do any classes or teachers stand out (in a good or bad way- but please do not include real names)??

- Why are you in 211? What are your goals in this class? How will you measure your own success? What can I do to help you achieve your goals? What can you do to help you achieve your goals?
Guiding Questions:

- Have you been bringing portfolio questions to class to share with your peer review group? Has your group formed a method for distributing papers amongst yourselves and sharing feedback?
- Can you name one thing you have learned or changed because of this feedback?
- As you think about the way you have been working with your group, is there anything you would change so that it can be more helpful?
Guiding Questions:

- After gathering all of your returned homework, look for any patterns in the comments. Are the comments aimed at higher-order concerns (like, fundamental flaws in the flow or structure, or mistakes in the mathematics) or lower-order concerns (like, format or mathematical notation convention)?

- After reading a proof from Section 4.1 or 4.2, then comparing with a later proof, what changes do you notice in your writing?

- Describe your writing process. After doing so, explain at what stage in the process you can make improvements based on the patterns you mentioned.
**Portfolio Details:** Each portfolio will consist of: (i) 10 problems (at least 6 must be proof problems) chosen from throughout the semester from among the recommended but uncollected and ungraded problems and (ii) a typed one to two page cover letter.

- Students did peer-review of each other’s portfolio problems, at the start of most class days.
- Portfolios were written in LaTeX, the typesetting program preferred by mathematicians.
Why Portfolios?

From the syllabus:
Throughout the semester you will be getting feedback on your work. On your homework, you will not just receive a grade but comments and suggestions, too. In peer review groups, you will be giving feedback to each other on your proof-writing. In your journal you will be reflecting on this feedback. The purpose of the portfolio is to draw on all of these components to provide a capstone piece which serves as evidence of your thoughtful reflections and showcases what you have learned in this course. The portfolio will include a cover letter that stands as a reflection piece on why you chose the problems you included and how they demonstrate your strengths and your improvement in the course.
Attend peer-review day in class

Assist groups with out-of-class peer review

Give a mini-lesson on peer review and mathematical writing

Meet with professor to share insights and observations about the class

Goals that we did not meet
  - Meeting with students early in the semester, in an on-going fashion
  - Additional ‘Mathematical Writing’ component
I think that the WA program as it currently stands is better suited for the humanities and social sciences, but with some alterations, it could definitely be a very effective tool in more technical, content-specific classes like 211. This semester was definitely a step in the right direction, as it was the first time a WA was placed in a math class. Because of the major shift from typical academic writing to math writing, I think it is important that the bulk of their instruction in that area does come from the professor, especially since it is so tightly tied to content.
Students engage in planning exercises.
Students write a proposal or prospectus.
Students revise one paper.
Students revise all papers.
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Reflections
What a Fun Class!

- We got to know the background, fears, and ambitions of the students. We got to hear their feedback on the course “in real time” and were able to provide timely guidance and support.

- The quality of the work in the portfolios is impressive. We believe the course components made this possible.

- We hope the students had a “successful” invitation to mathematics and mathematical writing.
Graded only on completion; collected 3 times per semester.

By attending the WR (and FYS) workshops and working with Noreen, we got suggestions for writing good prompts.

These journals are just one type of low-stakes writing; we’d like to add others.

The goal: transfer. Our questions: how do we measure success? How do we analyze data to show the goal was or was not met?
- Total weight: 15%; only the final draft was graded.
- Instructor feedback was given one-on-one in short consultations.
- Writing Associate lead a session on the peer-review process.
- We required portfolios to be TeX’d (pros and cons).
- This course portion requires a lot of planning!
- The goal: this capstone piece showcases mastery of mathematical content and written communication in our discipline. Our questions: does this work ‘better’ than a standard model? Would such a project be feasible in all classes? Why would this not work for you?
The Future is WAC
Writing Center and Writing Associates:

- Noreen’s across-the-disciplines assessment in AY 2011 of Writing Center usage rates revealed less than 1% of the 580 students enrolled in Mathematics used the Writing Center (six visits by two visitors).

- Since then, Noreen has been actively recruiting writing tutors who could also tutor proof-writing courses. In the fall, 4 members (of the staff of 70) will be qualified to work with Math students.
Strengthening Departmental Connections:

- Noreen has found many resources on the web and created an additional Mathematical Writing tab on her Writing Center page on Moodle.

- The Math Dept. faculty have reached out to Noreen in navigating the college FYS and WR requirements, and Noreen has been empathetic and supportive.
Discussion Questions:

- How do instructors and WAC proponents work together to decide on course components?
- How can instructors (especially in disciplines besides the humanities) overcome their anxiety about incorporating writing, both as an ‘end product’ and as a way of constructing knowledge?
- What do faculty need from their Writing Center?
- Besides journals and portfolios, are there other suggestions for how we can integrate writing into the course allowing for the unique challenges of writing (and revision) in a math class?
- We’d love to discuss the possibility of writing about our experience. Are there suggestions for how we can mine the available data?