The mathematics learned in college will include concepts which cannot be expressed using just equations and formulas. Putting *mathemas* on paper will require writing sentences and paragraphs in addition to the equations and formulas. —Kevin Lee, author of *A Guide to Writing Mathematics*

Introduction

At Wittenberg University, as at many schools, students are required to take several writing-intensive courses. Such courses are taught in all the disciplines and, according to the Faculty Manual, include “writing as an integral part of teaching and learning, with class time devoted to the discussion of the writing process and assignments designed to reinforce and develop writing skills.” Ideally, students in these courses should enhance their writing ability as they enhance their knowledge of the course content; the question, though, is how to handle these overlapping goals. In this article we describe a writing project that requires students to create textbook chapters for their peers—an assignment in which students must demonstrate their knowledge of the course content by sharing it in writing with their classmates. This textbook-writing assignment, or a modified version of it, might be applicable to other writing intensive courses across the curriculum.

One of the courses designated as writing intensive at Wittenberg is Math 210: Fundamentals of Analysis, a sophomore-level mathematics course that concentrates on the “study of functions, set theory, sequences, the real number line, logic and methods of mathematical proofs.” To integrate writing into such a course, a teacher can rely upon a good number of published suggestions. Students might be asked to write about a mathematician (Crisman) or to write letters to friends or relatives explaining what they’ve learned in the class (Goodman). Benadette Russek catalogs several different assignments that her department uses, assignments she places in “the category of writing...
about mathematics learning” (36). There are also excellent collections of assignments such as Annalisa Crannell et al’s *Writing Projects for Mathematics Courses: Crushed Clowns, Cars, and Coffee to Go* that offer students a chance to use their mathematical knowledge “by presenting them with a reason for responding with a clear written solution” (2). For example, they might need to respond to a letter from a greenhouse worker and help him “find an equation for the curve of a leaf” (13).

In 2006, when teaching Math 210 for the first time, Adam Parker allowed the students to choose to write about either a mathematician or a mathematical theory not covered in the class text. These writing projects were 10–20 pages long, and students were required to draft and revise based on comments from Parker and their classmates. On the one hand, Adam found these projects valuable. The writing was appropriate for sophomores who had taken introductory English writing classes as freshmen, and he usually saw an improvement from first drafts to final versions. However, there were also serious problems. Topics proposed by the students were often too broad and unwieldy in presented forms, and the topics had to be pared to such an extent that the students lost interest. Also, when students chose a “famous” problem, they immediately found that these problems were usually not just hard to solve, but also to explain. Adam attempted (with varying success) to overcome these issues by scheduling multiple early meetings to make sure students were on an appropriate track.

Yet, he was unable to overcome the students’ perception that these projects were not well integrated into the course. The content of the course consisted primarily of mathematics—writing, editing, and revising proofs—but then the writing project consisted of no proofs and little mathematics. As a student mentioned in the final evaluation, “[T]he ‘writing intensive’ qualification seemed tacked on and dumb—this is supposed to be a math class right?”

Mike Mattison has also had experience with such assignments—in his role as a writing center director—and he has heard similar complaints from students. They claim the assignments are busywork and do not really relate to the class material. Their eyes roll when they talk about the writing; the students have not always made the connection between the assignment and the course work, or they simply are not interested in what they are writing about. The paper is something they had to do in order to receive “writing credit” for a course. It’s a game and everyone is playing along. Even the literature cited above mentions similar difficulties with such work. For Goodman’s letter-writing exercise in a calculus class, “[m]any students were ambivalent” about the assignment, finding it “merely another assignment to complete” (4).
We feel ambivalent, too. At the core of the ambivalence is the knowledge that these projects do not, in general, balance writing mathematics and prose. To return to our epigraph from Lee, the emphasis in many of these assignments is on the paragraphs and sentences, but those are composed separately from the equations and formulas. The assignments are asking students to write about mathematics; they are not asking them to write math. As much as the assignments appear to give students a way to apply mathematics to the real world, they still do not necessarily ask them to enhance their mathematical skills. The students are not engaging with the symbolic language that mathematicians use; they are not writing math.

Writing Math
Mathematics professors face challenges when constructing discipline-specific writing projects that are unified with the flow of a course. Perhaps the most obvious is that writing mathematics doesn’t just involve using English words to write in the subject (as perhaps might be done in the humanities or social sciences.) Rather, mathematics involves learning an entirely new symbolic language that has evolved over hundreds of years to be concise, precise, and extremely dense. Teaching this language is essential to any writing course in mathematics. As noted by Flesher, “[T]he concept of writing to learn in mathematics should be learning the new language of the new discipline” (37).

From the start of mathematics until the early 1400s, mathematicians wrote out all of their work in sentence form. This was an extremely cumbersome process that inhibited advances in the field (Kline). Mathematicians realized the limitations of the rhetorical nature of this communication and started (initially in an “incidental” or “accidental” way) to symbolize the subject.

A systematic symbolization started with Vieta (b. 1540, d 1603), Descartes (b 1596, d. 1650), and Leibniz (b. 1646, d. 1716). By the end of the 17th century, mathematicians had embraced the power of notation, an embrace that led to the expansion of algebra, number theory, calculus, geometry, etc. As Flesher notes, “All formulas have verbal meanings that are analogous to the translation from one language to another and work as a glossary” (38), and the savings in clarity and space are often dramatic.

Burger and Starbird, in The Heart of Mathematics—An Invitation to Effective Thinking, illustrate the power of mathematical notation. This text is designed for non-mathematics majors and touts itself (accurately) as a guide to “some of the greatest and most interesting ideas in mathematics” (ix). One such idea is Cantor’s celebrated Power Set Theorem. To describe the theorem and its proof takes the authors seven pages (175–181). At the end they write:
Just for fun, we will now write down the exact same proof using the extremely abbreviated, cryptic notation that makes mathematics succinct but difficult to read. To unravel and understand this one-line proof, a reader would have to produce the preceding explanation. We write it only for your amusement, so we will not bother to clearly define the terms or explain the notation.

Consider \( f : S \to P(S) \). Then \( \{ x \in S : x \notin f(S) \} \notin f(S) \). Q.E.D.

This one line encompasses several thousand words of deep and important reasoning, leading the authors to warn that in mathematics, “Length is not a good measure of depth” (183).

An important goal for a mathematics professor is to have students understand what lines such as the one above mean, as well as how to construct meaningful proofs from this “extremely abbreviated, cryptic notation.” A professor should require students to use this language, develop it in every single mathematics course, espouse the power of this language, and utilize many writing techniques (editing/revision/drafts) in order to make sure that students can express themselves using it.

So, how can we create an assignment that is integrated into the course in a meaningful way and that helps students write math instead of only writing about math? One possible avenue is to have students write in a genre that requires both: a textbook. Textbook authors need to write the proofs and formulas required of mathematicians, but they also need to write about the process, explaining the concepts to their audience.

**Assignment**

For the spring 2009 semester, Adam taught Math 210 for the second time. The course is required of all math majors and minors, and each section has between 15–20 students, half of whom are majors and half minors. This is a “bridge” course that is designed to prepare students for the rigors and abstraction of upper-level courses.

The assignment began after the sixth week of class. At that point, the class had covered roughly six sections of the text and had taken a first exam. After the exam, Adam gave the students a chance to evaluate aspects of the course, specifically asking for feedback on the textbook. Based on those responses, the students and Adam discussed what made a good or a bad textbook; they talked about textbook pedagogy (this conversation appealed to the students wanting to be math teachers). As a mathematics professor, Adam could predict the student comments. The students desired extremely clear proofs with absolutely no jumps in logic, and they certainly didn’t want any proofs that were “left to the reader.” Students wanted many examples and exercises of varying
difficulties, with solutions to the exercises presented. They wanted pictures, a review
section, and prose that gave the ideas of the theorems in “everyman terms.”

The class agreed that these qualities should be present in any good textbook, and
Adam used this discussion to motivate the assignment: students would write a textbook.
Or, more precisely, each student, working either individually or in a group of up to four,
would rewrite a section of the course text. (See Appendix A for the assignment sheet.)
Beyond that general guideline, there were few limitations. Adam intentionally left the
assignment vague as he wanted the students to really think about what they would want
out of a textbook. The requirements:

• Smaller sections are reserved for those working individually, larger ones for
  larger groups.

• Every section must include at least three clear and complete proofs, with more
  if the students feel it makes a better textbook.

• Most mathematics textbooks have questions at the end of each section to give
  students practice in applying concepts from the chapter. Each group must create
  five such problems for the section, along with complete and detailed solutions.

• If the group decides to diverge significantly from the “good practices” that
  were discussed in class, they should provide justification (as to why they didn’t
  include examples, for example).

The choice of whether to work together or alone was left to individual students.
Those deciding to work alone cited a variety of reasons, from difficulty in finding free
meeting times because of work or athletics to a preference to have complete control over
their final grades. (The individual projects were of a bit higher quality than the group
work, a point to consider for future versions of the assignment.)

The students worked on the project for the remainder of the semester. While
the class continued covering chapters seven through fourteen in the textbook, the
students were also drafting and revising one of the first six chapters. So, each homework
assignment from a subsequent chapter also had some aspect of the project due at the
same time: outlines, division descriptions of group responsibilities, individual rough
drafts, group rough drafts, edited drafts, and final versions. (See Appendix B for a more
detailed timeline.) These writing assignments were graded and turned back quickly so
that students could incorporate comments and suggestions into the next part of the
project. The final version was due two weeks before the course ended. After the projects
were turned in, students filled out two evaluations of the course. One was an anonymous
assessment of the writing project itself (see Appendix C). The other evaluation was signed and was designed to ascertain how evenly work was distributed in the groups (see Appendix D).

After being turned in, the projects were immediately graded for accuracy on the mathematics, and then Adam made copies of all of them in order to distribute them to the class. Students thus had alternate versions of the first six chapters of the text, as well as a set of practice problems (with solutions); they were encouraged to use these materials to study for the final. During final exams, Adam graded the writing content of the projects. In total, the assignment was equal in weight to a midterm exam, which amounted to 13–18 percent of the course grade. The grade was based on the intermediate assignments, group participation, and the mathematical and English prose writing in the final version.

Results

As we’ve tried to make clear, the hope for this assignment was that it balanced an emphasis on the writing process with an emphasis on writing the content of the course—writing math. According to both the students and Adam, such a balance seems to have been achieved, at least to some degree. In the anonymous survey, every student believed this project was a positive writing experience for a math class. Adam found that the project allowed students to display their knowledge in a more coherent fashion, and the results also offered him some additional teaching strategies.

The assignment was unquestionably a writing project. Significant time was spent discussing good writing practices such as brainstorming ideas, creating outlines, writing and revising rough drafts, and editing. Students were encouraged to revise and edit within their group; they were also encouraged to exchange drafts of their projects with another group and to visit the Wittenberg Writing Center in order to obtain additional feedback from readers. From their comments after the class, it appears that students appreciated having deadlines to keep them on track, and they appreciated getting feedback from Adam throughout the semester. In fact, one student suggested starting the project earlier so that there would be time for more revisions and feedback. The students also began to consider the constraints of the textbook genre and how writers must work to communicate with their readers in this form. Said one, “Writing this project made me think about how people actually learn and [about] the difficulties in being concise yet understandable.” Another said that the project made her “think about the importance of phrasing and word choice when trying to make a concept clear.” And Adam saw an improvement in the writing from the start of the project—students were making strong revision choices.
Furthermore, the students themselves believed that the writing they were doing was helping them understand the course content. They were not only considering their rhetorical choices; they were learning the material. Of the 13 people in the class, every single one felt that the project helped with understanding the topic of the chapter. It was a matter of learning the material well enough to be able to share it with another: “I had a much better grasp of some topics after doing this project. Having to look deep enough into the subject to feel that I could teach it helped a lot.” Many students mentioned how helpful it was to have to design problems and create questions: “I definitely have a more thorough understanding of the topic and I think simply having more practice problems and considering how to re-word information for clarity was most helpful.”

But, did the students’ work align with their beliefs? Adam found that the students did seem to display a better understanding of the mathematical concepts in their writing. For example, one subtlety that can be difficult to grasp is that mathematics isn’t necessarily absolute, but rather that the truth or falseness of a statement depends on the context of the problem. Multiple groups tackled this topic in their chapters, and their descriptions were clear and straightforward:

For the time being, if no system is indicated, you should assume you are working with the real numbers. This assumption will be necessary when working with quantifiers because if you are trying to prove something for all arbitrary numbers in some system, you need to know what that system is.

For instance, the statement:
There is only one value of x such that $x^2=1$ is true for the natural numbers (the positive integers not including 0), but is false for the real numbers because both 1 and −1 squared would equal 1.

Or consider this example:
You may think that this definition of complement is overly complicated. Why didn’t we just define “$\bar{B}$” to be the set $\{ x : x \notin B \}$? The problem is that $\{ x : x \notin B \}$ is too large. For example, suppose that $B=\{2,4,6,8\}$. Then $\{ x : x \notin B \}$ would contain all of the following things (and more!):

- The integers 1, 3, 5, 7, 9, 11
- The real numbers greater than 25
- The function $f(x)= x^2+3$
- The circle of radius 1 centered at the origin
- The Empire State Building
- The state of Minnesota

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It is quite reasonable that the integers 1, 3, 5, and 7 should be included in \( \tilde{B} \), and depending on the context we might want to include the real numbers greater than 25 as well. But it is quite unlikely that we would want to include any of the other items. Certainly, knowing that the state of Minnesota is not a member of the set \( B \) would contribute little to nothing to any discussion of \( B \).

What strikes us about these examples is that the students are not just presenting the definition or notation, but rather that they are explaining why these are the correct concepts. Both of these examples use a rather subtle logic technique where they “suppose” we were to use a more naive definition, and then they construct some absurdity from that supposition, hence concluding that the more naive concept cannot be correct. In the first case, if we didn’t provide a context for our problems, we would end up with ambiguity in the validity of certain concepts. In the second case, the authors hypothesize a different definition of *complement* and show that it leads to absurdly large and unhelpful sets. These examples show not only an understanding of the definitions but also a grasp of why these are reasonable definitions; and the students are making strong, logical arguments.

This student work represents for us a good balance between sentences and paragraphs and equations and formulas. An additional benefit of the assignment was that creating proofs and explaining them helped students write better proofs on homework, quizzes, and exams. Although Adam did not do a systematic study of the performances on the final to determine if students did better on the material from their rewritten chapters, he did note that overall performance on the final was better than in the previous year. Of course this could be for many reasons, but we do feel this project contributed at least in some way to the improvement.

One of the reasons we believe the assignment worked well was that the students knew they were writing not only for Adam, but for each other. As the assignment sheet read, “I’ll grade the papers for mathematical correctness, make copies of all projects, and distribute them to the class. That way you have other people’s chapters and questions to use to study for the final.” The students had a well-defined, real audience for which they were writing. As we noted earlier, projects can have a wide range of audiences—a family member, the teacher, the local greenhouse worker. However, as Kiefer and Leff argue, “Even those assignments with specific rhetorical contexts or designated publication guidelines get interpreted by students as merely academic exercises” (para. 1). It is difficult to convince students that they have an audience other than the teacher. Here, though, the assignment did not aim for an outside audience or a specific publication.
It was geared towards helping students study for the course itself. The students knew exactly how much mathematics the audience knew and therefore exactly what level to write at. All students were familiar with textbooks and so could intelligently create a chapter for another student. Most importantly, they knew that their peers were going to be reading, studying, and using their projects. The students had become the teachers, and they seemed invested in this project.

At the same time, not every student believed that using the rewritten chapters was beneficial. In contrast to the unanimous agreement that working on their own chapter contributed to their learning, only eight of the 13 students said they found it valuable to use other groups’ projects to study. We think a contributing factor to this response was that the survey was taken on the last day of class, and some students hadn't yet started studying for the final. Regardless, it will be a goal in the future to stress this opportunity to the students because we feel this is a genuine benefit of the assignment. For those students who did report a benefit, the comments concentrated on the advantage of having more examples and hearing an explanation from another perspective: “More practice problems and seeing proofs worked out was very helpful” and “Seeing the proofs worked out again and more practice problems with solutions is always helpful.” The authors agree, especially when the instructors don’t have to make up the problems and solutions.

Adam’s teaching also benefitted from the projects. Some of the students really thought deeply about how they learn and therefore incorporated wonderful pedagogy into their projects. Some of the ideas were easy, simple, and obvious ways to improve the classroom experience for a student. For example, one group listed times next to every example and problem in their chapter. These times gave the other students an idea of how fast they should be able to do the problems for a test. This made so much sense! Often students take their time doing homework, and then feel rushed on an exam. By providing something as simple as a goal time, a textbook can better prepare students for tests. Another student, when creating problems for the end of her chapter, decided to include exercises that drew from various sections of the book. She noted, “It’s easy to do problems when you know what techniques to use. But you mix things up on a test and don’t tell us how to do them. So I wanted to give practice at that.” She noticed that learning is cumulative, and incorporated that idea into her sample chapter. There were multiple improvements to Adam’s teaching gleaned from what the students felt was valuable to their learning. It was particularly pleasant to see these suggestions coming not from the literature on teaching, but from the students themselves. We predict students will continue to generate these teaching ideas as this assignment is given to future classes.

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Challenges and Changes

Given the results from the first attempt, as well as the student responses, Adam will definitely use this project again, both for Math 210 and other writing-intensive courses he teaches. However, the project will be changed slightly based on his own observations and on suggestions from students.

There was one consistent comment from the evaluations. Since the choice of chapters was left up to the groups, the projects were not uniformly distributed; some chapters were not rewritten and others were rewritten by several groups. Even worse, the rewritten chapters tended to be the easier sections. Students thus wanted the chapters assigned to groups, and Adam will be doing just that in future years. A few students also requested a more detailed grading rubric at the beginning of the project, and such a rubric will be provided for future classes.

The project has two other items to watch for as an instructor. First, there will be the standard group dynamics that must be dealt with. These dynamics are endemic to any group project—people may not get along and some people may not pull their weight. This lack of participation may be because they aren’t willing, or because someone else is dominating the group. It’s up to the professor to determine how to deal with this. Adam provided a feedback sheet that gave people the opportunity to comment on the workload distribution, and these comments were taken into account for the grades (see Appendix D). At the end of this evaluation, he asked if the students would prefer to have him assign the groups and/or assign topics to the groups. Of the 13 students, only three wanted assigned groups. Very few wanted assigned topics, although, again, they did want Parker to make sure there was no “doubling up” of the chapters.

Typesetting the symbolic language of mathematics can also be a challenge. Microsoft Word has an equation editor that can be used. It is easy to figure out, though it has limited symbols and can take a long time to input formulas. Adam encouraged some of his students to learn and use the mathematical typesetting program LaTeX. This program is the way that almost all papers in mathematics, physics, and computer science are typeset—it is the “industry standard.” Some students were familiar with the computer algebra system Mathematica, for which Wittenberg has a site license. It is possible to write papers, presentations, posters, etc. within this program, and inputting mathematics is easy, so Adam allowed use of that program. While each of these options will easily produce typeset formulas and text, the inclusion of graphics, shapes, diagrams, etc. is not as easy. The solution that Adam came up with was to let the students type all of the prose using whatever program they wished, but they left space for diagrams or complex mathematical notation and wrote those in by hand. The
handwritten sections did give the final projects a certain late-1800s feel, but the students didn’t seem to mind.

**Conclusion**

Again, a major concern with writing projects in mathematics (and other courses as well) is that they often feel tacked on and artificial. This feeling can result in unhappy students and ineffective projects. We are encouraged that students felt that Adam’s project integrated naturally with the course. It appeared that some students came into the Math 210 with a preconception of what the writing project would be like: one of the standard “writing about math” projects described in the introduction. Two students commented that the textbook project was “better than writing about a single proof” and “much better than a history paper.” Considering these were exactly the alternative projects Adam was contemplating, he was happy that he chose to give the students this new assignment instead.

And we believe that this type of assignment can be adapted to other courses and disciplines. Students could just as easily be asked to write a section of a textbook for an art history class or a chemistry class or a psychology class or a geology class. In each of those cases, students will need to wrestle with the material and with ways to communicate that material effectively to others. If the textbook sections are given to the class members for study guides, then the students have an engaged, interested audience that has a stake in the material.

Granted, this assignment has been offered only one time. It is, certainly, a work in progress, as most writing assignments are—we should revise our work just as students should revise theirs—but the promise is clear. The assignment balances “writing about math” with “writing math,” and, for the students, this balance comes in a project that they find relevant and useful: “[I]t’s difficult to come up with a good writing project (I know, I’m going to be a math teacher too!) but I think this was very valuable, creative, and a great review technique.”

**WORKS CITED**


Kiefer, Kate, and Aaron Leff. “Client-Based Writing about Science: Immersing Science Students in Real Writing Contexts.” Across the Disciplines 6 (2009). Web. 20 Nov. 2009.

**APPENDIX A**

Math 210- Fundamentals of Analysis

Writing Project – Spring 2009

Working individually or in groups, you will be re-working a section of the text that we’ve already covered (Sections 1–7 and the Pigeonhole principle). Your job will be to improve the chapter, by incorporating things that you find valuable in a textbook. You should think about the discussion we had after the first exam for some examples of good practices. I would think most of you would have many good examples, pictures, proofs worked out in complete detail etc. Think deeply about how you learn and what you would want from a good text.

**Requirements:** I am intentionally leaving this assignment open ended because I want you to be very creative in how you design your chapter. The few requirements are:

- The smaller sections (such as Pigeonhole principle, Section 2) will be reserved for smaller groups.
- Every section must include at least three complete proofs. There should be more if you feel it makes a better textbook.
- Each group must create at least five new questions, and write up complete and clear solutions to them.
- If you chose to do something very different from what we talked about in class (for example, if you decide you like textbooks with no examples) you should provide justification, so I know you’re not just being lazy.

There is no page requirement, though I think you should easily triple the length of the chapter in the text by adding sufficient details to their arguments.
Grades: This assignment is both a mathematics assignment and a writing assignment. This means you’ll be assessed both for mathematical accuracy and on both your math and English writing. To get a good grade, you’ll have to do both well.

Grades will be broken down using the following rubric.

- Group memberships with chapter choices 2%
- Description of labor with detailed outline, bibliography 6%
- Individual Rough Drafts 20%
- Group (Unified) Rough Drafts 20%
- Final Project 40%
- Group Participation 12%

The project will count either 18 percent or 13 percent of your final grade, depending on which grading scheme we use to calculate your final grade. This is the same as a midterm exam.

Due Dates: Intermediate assignments will be due concurrently with homework as we move through the semester. The final project will be due on April 29. At this point, I’ll grade the papers for mathematical correctness, make copies of all projects, and distribute them to the class. That way you have other peoples’ chapters and questions to use to study for the final.

APPENDIX B

Timeline for Writing Project

This is a timeline for the progression of the project. Those entries marked with an * indicate a graded portion of the project.

- Week 6 Assignment given
- Week 7 Group members and chapter choices due*
- Week 8 Rough outlines due
- Week 9 Detailed outlines w/ bibliographies due*
- Week 10 Nothing due—working on drafts
- Week 11 Rough draft due*
- Week 12 Rough draft due*
- Week 13 Final version due*
- Week 14 Graded for mathematical correctness
- Week 15 Distributed chapters to class to study for final
- Finals Week Graded for writing
APPENDIX C

Writing Project Evaluation

GENERAL QUESTIONS ON PROJECT—These questions are just meant to give me feedback about this type of project (writing a chapter of a text). Oftentimes writing projects in math classes can seem artificial, and I’m constantly search for ways to make them more natural. This part is anonymous.

1. Did this project help you appreciate the work that goes into writing an actual textbook?

2. Did this project help in understanding the topic of your chapter? What was the most helpful part (seeing the proofs worked out, different presentation of the information, having more practice problems to work on etc.)

3. Did using the other projects help in your studying for the final? What was the most helpful part (seeing the proofs worked out, different presentation of the information, having more practice problems to work on etc.)

4. I intentionally left much of the interpretation of the project up to you. Would you have appreciated more formality and direction?

5. Overall, was this a valuable writing experience for a math class?

6. How should this project be changed, scrapped, or tweaked?
SPECIFIC QUESTIONS ON PROJECTS—These questions are meant to give me feedback on how your group worked together on the project. These questions can’t be anonymous, but they won’t be shared among other members of your group or the class.

Name:

1. Do you feel that the work done on your project was evenly distributed through the group?

2. Were all ideas taken into consideration when the group was deciding what to include or how to proceed?

3. Please distribute 10 points (total) among your group based on how much you felt each member contributed. Feel free assign non-integer points as long as they add up to 10.

Name: number of points

4. In the future, should I assign groups and topics?