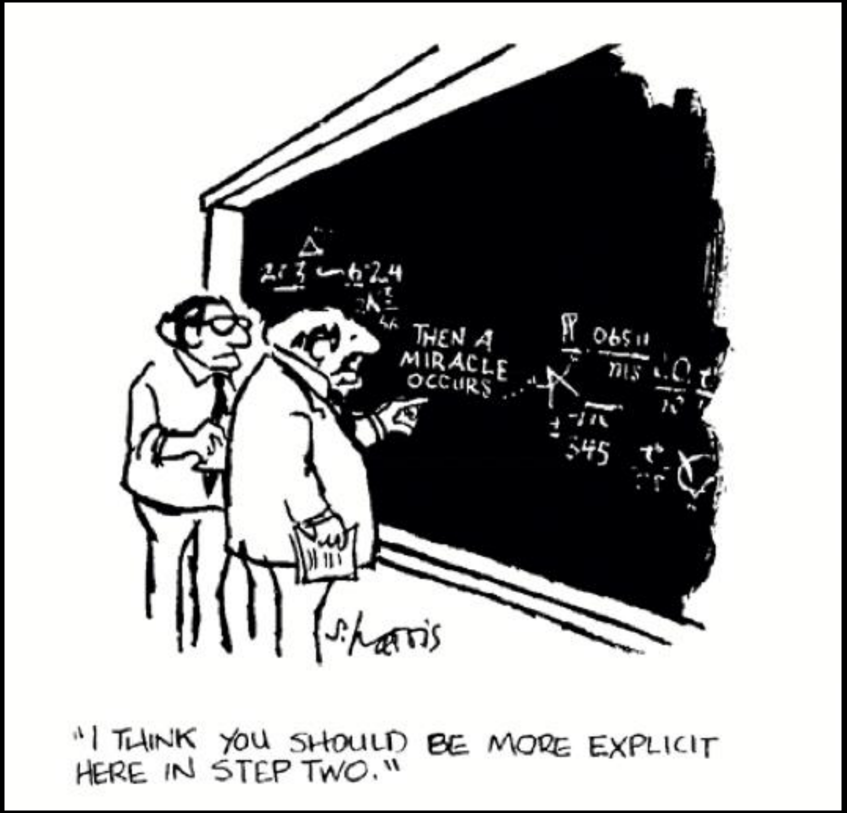
Writing in the Discipline of Mathematics

Writing can be incorporated into most of our mathematics courses – in particular upper level courses.

We combine writing to learn assignments with the more product-focused learn to write pedagogy of writing in the discipline.

Goal of the Writing Projects

- To synthesize the course material through research and creativity
- To further your experience in writing correct, concise mathematical proofs, while investigating mathematical issues



Low-stake/high-stake Writing

- The low-stakes writing to learn assignments help students to comprehend mathematical concepts and to become more fluent communicators.

The high-stakes writing in the discipline project teaches the students the structure and rhetorical form of writing in mathematics.

Low-Stake Writing Assignments

In these assignments, the students are writing to learn. They spend time regularly reflecting on their writing to make sure what they are presenting is clear to their own classmates and not just to the professor.

When students write in a low-stake format, they must organize the mathematical concepts in a logical order and present them in their own language.

Individually, these types of writings don't affect the

Different types of Low-Stake Assignments:

There are several types of low-stake writing assignments that can be used in mathematics courses.

- Reading Questions
- Proofs
- Group projects
- Summary of colloquium talks

- Reading Questions

In this assignment, the students are instructed to read the next section of the book to be covered by the instructor during the next meeting period and be prepared to write about the new concepts that are to be discussed.

Sample example

Reading Questions Section 1.9

Read all of Section 1.9 carefully and work through the examples presented. Then, answer the following questions:

1) Suppose we are in R^n. Define what is meant by $I_n, e_1, e_2, \ldots, e_n$. You may need to browse through other sections to determine this information.

- I_n is the $n \times n$ identity matrix. That is the matrix with 1's on the diagonal and 0's elsewhere

$$\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

- e_j is the vector in $\mathbb{R}^n$ with a 1 in the jth entry and 0's elsewhere

$$e_j = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow \text{jth entry}$$

2) State Theorem 10.

Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then there exists a unique matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^n$.

In fact, A is the $m \times n$ matrix whose jth column is $T(e_j)$:

$$A = [\, T(e_1) \;\; T(e_2) \;\; \ldots \;\; T(e_n) \,]$$

- Proofs
- Proofs are by far the most difficult writing assignments in math courses because:

- In order to construct a proof which makes sense, a student must have a clear understanding of the preceding material, and fully comprehend what is to be proved.
- The students must carefully construct a sequence of logical statements.
- Each statement must be precise and be a logical consequence of previous statements or other assumptions.

Example

Indicate if the following statement is True or False. If it is true, give a detailed proof and it is false, give a counter example.

If A is an $n \times n$ matrix, then the homogeneous linear system $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution **iff** det(A)=0.

Proof: This is an "if and only if" statement so we must prove both assertions in order to show that the statement is True.

"$\rightarrow$": We assume that homogeneous linear system $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution $\mathbf{x} \neq \mathbf{0}$, and show that det(A)=0, by "proof by contradiction".
If det(A) $\neq$0, then by a previous lemma, we know that A^{-1} exists. Now multiply both sides of the equation $A\mathbf{x} = \mathbf{0}$ by A^{-1}; $A^{-1}A\mathbf{x} = A^{-1}\mathbf{0}$. Since $A^{-1}A = I$ and $A^{-1}\mathbf{0} = \mathbf{0}$, the equation $A^{-1}A\mathbf{x} = A^{-1}\mathbf{0}$, becomes $A^{-1}A\mathbf{x} = I\mathbf{x} = \mathbf{x} = \mathbf{0}$. But this is a contraction as we know $\mathbf{x} \neq \mathbf{0}$.

"$\leftarrow$": We assume that det(A)=0 and show that $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution. Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ be a solution for this linear system. We know by a previous lemmas that if det(A)=0, then the last row of RREF of the augmented matrix $[A \mid \mathbf{0}]$ consists of all zeros which implies that there is a free variable, $x_n = t$, and the rest of the variables, $x_1, x_2, \ldots, x_{n-1}$, can also be determined in term of t. Therefore, there are infinitely many solutions. QED

Some student responses

1) True: A is singular and A^{-1} D.N.E.

2) True: Det(A)=0. Singular. So has nontrivial solution.

3) True: It can be parameterized to infinite solution if det(A)=0

4) True: **Ax=b.** 0 or ∞ many solution based on **b**. A^{-1} doesn't exist. A is singular. **Ax=0** has no solutions other than **x=0**

5) False: Lineally dependent: Combination

6) False: Only non-trivial solutions if A has an inverse and A can only be invertible if det(A) $\neq 0$

7) True: If det $\neq 0$, only trivial solution. $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ x+2y=0 y is arbitrary.

8) True: If A is nonsingular det(A) $\neq 0 \therefore$ det(A^n)$\neq 0 \therefore$ A^n is nonsingular.

9) False : det(A) $\neq 0$, A is not invertible.

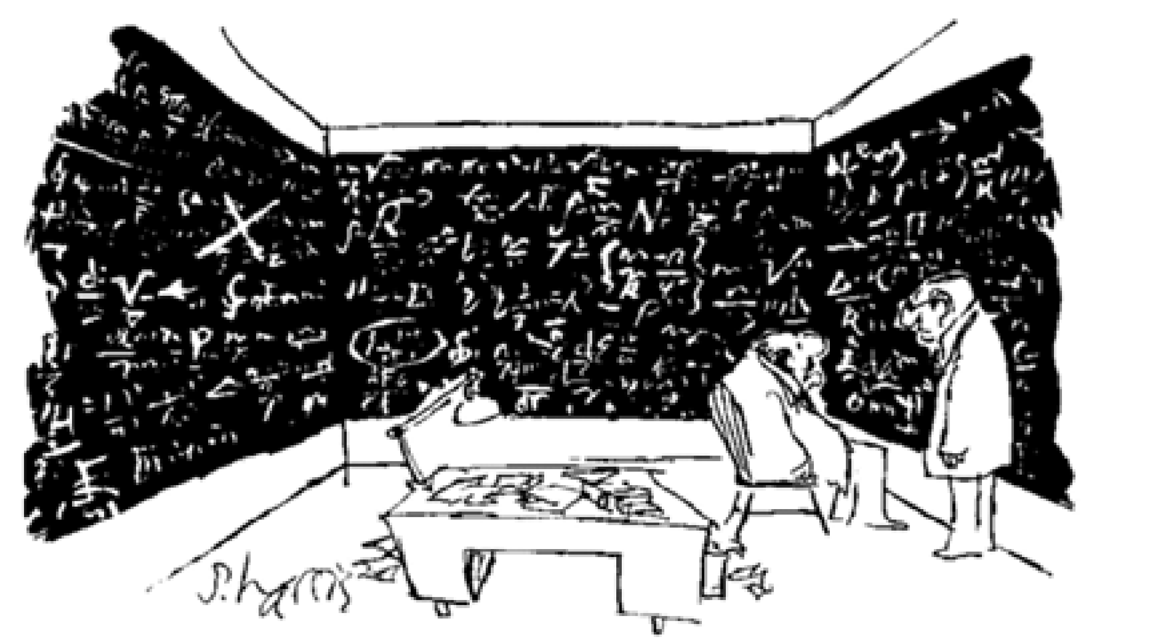
This student changed **iff** to **if** by crossing out one **f**. He thought it was a misprint.

10) True: Looks reasonable.

- Group projects

The students will work together on a group project (usually a computer project). They are required to turn in their results with an explanation and interpretation of the outcome.

Often students run the project and find the results but are not able to explain their findings.



"Whatever happened to *elegant* solutions?"

- Summary of colloquium talks

This writing assignment encourages students to participate in the colloquia we offer in the department of mathematics on a regular basis. The students will receive extra credit by attending these talks and writing a one-page summary of the speaker’s presentation.

Sample

CAM Lecture – Cell Movement

Mathematics strikes again in the human body. Molecular biology continues to be affected by the growing field of applied mathematics, and Ryne Smith showed us how. Ryne Smith is a St. Thomas alumn who has found success applying his passion for mathematics to his other passion: biology. Under direction of Dr. Stolarska, Smith has developed ways to model cell growth and cell movement in the human body. Specifically, the two worked with Dictyostelium Discoideum. Their efforts have allowed progress to be made in cancer research and allowing scientists to better understand how baby fetus's develop.

Our bodies are made up of millions and millions of cells. Everyone knows that. But so much more is yet to be discovered about how these cells interact with each other. Cells move throughout our body by expanding and contracting. Constantly pushing against one another, they migrate around our bodies. Ryen Smith was charged with the responsibility of trying to model these movements and interactions with a computer. Very quickly it became clear this was not an easy task. One has to take into account countless forces acting on a cell and all of these associated repulsive forces that result from just one cell movement. Smith examined groups of cells and created a mesh-like representation which assigned "nodes" to points in the groups and complex computer algorithms could track these nodes to see how they behave compared to other selected nodes.

There is a significant amount of background theory needed to understand much of Smith's findings, but he discussed Young's modulus and the Poisson ratio. The first is a measure of elasticity of a cell, and the latter describes a ratio between the possible stretch/compression of a cell. Dictyostelium Discoideum was found to be hypoelastic. The closest approximation they were able to conclude on was that a Dictyostelium Discoideum cell takes about 30 seconds to completely extend, and 10 seconds to completely contract. Other findings are hard to explain without visual representations, but it was clear that had made progress in predicting how many masses of cells would interact over time.

Mathematics applied in such a manner can allow scientists to predict how infectious cells can migrate and spread. This will allow medical companies to become more proactive in a fight against disease. It also became quite clear that there is significant room for growth in this field. Much is still unknown about cell interaction, especially considering that Smith's findings only correlated directly to Dictyostelium Discoideum. It was, however, a very interesting reminder as to how important math is and computational mathematics will become in our future.

High-Stake Writing Assignments

In this assignment, students are learning to write in the discipline of mathematics. They apply the mathematical knowledge they have learned in the course together with the analytical skills they have developed to write a paper on a topic related to the course. This paper should be directed to the other students in the class.

These types of writings usually worth 10-15% of the total grade.

The high-stake writing project can be broken down into a series of stages:

- Selection of the appropriate topic for the paper

Feedback: The instructor must approve the topic after reviewing the topic.

- Understanding and analyzing the mathematical concepts used in the paper

Feedback: The instructor will give feedback on the mathematical concepts

- Writing the first draft

Feedback: The instructor will give feedback on the first draft

- The final draft

Feedback: The instructor must approve the final draft before submission

Guidelines for the paper

• Paper Title • The Abstract • The Introduction • The Body • Examples • The Conclusions • Bibliography	• The paper should be five-six pages long. (single space, and 12 point font). • It doesn't have to be a masterpiece but It must be correct and have correct content and equations. • It should be typed (including the equations)

Sample papers are provided

Rubric for grading the paper

Each grading criteria has 10 points.

Grading criteria	**Points**
Mathematically correct	
Written in a Logical order	
Provides evidence/proof	
Clearly written	
Discusses the main result/focus	
Visually appealing	
Connects to the topics discussed in the course	
Total	