Proofs and Persuasion: A Cross-Disciplinary Analysis of Math Students' Writing

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Abstract: This article offers an initial analysis of the rhetorical devices used by mathematics undergraduates as they begin to write research articles in their discipline. The authors (a mathematician and three experts in composition and rhetoric) identify several such devices, including transitions and metacommentary, style and tone, use of sources, and visual rhetoric. Further, the authors use these markers, along with their unique disciplinary experiences, to identify convergences and divergences between writing in the disciplines of mathematics and composition and rhetoric.

Historically, rhetorical studies of college students' academic writing have focused on writing-intensive disciplines in the social sciences and humanities (see Afful, 2006; Beaufort, 2004; Stockton, 1995). Furthermore, writing textbooks designed to expose students to writing in other disciplines have a tendency to avoid actual disciplinary writing in math and sciences, including instead materials written about math or science topics for a popular audience or essays written about math or science education.[1] Since these essays reflect a disciplinary style more like that used in the humanities, students are not typically exposed to disciplinary writing in math. What research there is on math and writing tends to focus on writing-to-learn pedagogy and whether writing helps students learn mathematical content (see Clarke, Waywood, & Stephens, 1993; Shield & Galbraith, 1998; Porter & Masingila, 2000). Such studies are valuable in that they articulate how to use writing in order to learn mathematical reasoning, but there is little emphasis placed on how students might productively be taught how to write in the discourse of math scholarship. Burton and Morgan (2000) examine research papers written by academic mathematicians and identify different strategies writers use to establish their authority as professionals. They argue that their study has implications for the classroom; however, they concede more work needs to be done on identifying the academic writing conventions used by both professionals and students:

> In this exploratory study we have identified linguistic means for achieving various types of authority, significance, interest, and so on. But characterizing the various forms used by mathematical writers requires further research. This study could be a starting point for work with novice (and, indeed, experienced) researchers to develop their critical linguistic awareness–their knowledge of the forms of language that are available to them and their abilities to make effective choices among them. (p. 451)

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This paper takes up Burton and Morgan's invitation by examining, from different disciplinary perspectives, undergraduate mathematical writing. Specifically, we present the initial findings from an assessment of mathematical writing by students in a summer Research Experience for Undergraduates run at the University of North Carolina, Asheville. (This program is described more fully in a later section.) We have examined the scholarly writing produced by the sixteen students who participated in this program in 2008 and 2009 with the goal of identifying the rhetorical devices the students employ and axes along which we may trace students' development as writers in the discipline of mathematics. Below we will discuss our findings, offering divergences and convergences with the rhetoric of other, more widely studied, academic discourses.

We believe our work has several important consequences, both for classroom teachers and for those interested in interdisciplinary collaboration. Very little has been written about how mathematics students learn to write in their field, nor how mathematical writing might differ from the decontextualized "academic writing" taught in many first and second-year writing courses. This becomes a more pressing problem as mathematics students move out of secondary and lower-level university classes to upper-level courses in their major. Our work will assist instructors of upper-level math courses by highlighting those aspects of mathematical writing on which writing instruction should focus. This highlighting will help these math instructors to give more intentional writing instruction to their students. Furthermore, our work will inform scholars of composition and rhetoric as they struggle to understand what it means to "write mathematically." We are certain that our remarks on the rhetoric of math writing will enable WAC specialists, writing center directors, and teachers of science, engineering, and other mathematics-related fields to offer more robust instruction.

In "Why Assessment?," Gerald Graff notes the problems our classroom isolation from other disciplines can cause for students: "since [most] students fail to detect the common practices of argument and analysis that underlie their diverse courses, they tend to form a greatly exaggerated picture of the differences among faculty members, disciplines, and course demands" (p. 159). Graff insists that academics have much more common ground than they realize, noting that after all, there must be "tremendous implicit agreement in our practices, in how the academic intellectual game is played," otherwise we would not be able to "disagree with one another intelligibly or communicate our disagreements if such agreement on practices were not already in place" (p. 162). Although Graff acknowledges that he doesn't wish to downplay the very real differences between disciplines, it would seem to be worth finding out just how much common ground there actually is between two very different fields such as composition and rhetoric and math.

As stated above, it is our goal to highlight some of the disciplinary convergences and divergences between these two fields. We will start by giving background on the program and methods used to evaluate the papers, then consider the disciplinary similarities and differences that emerged from our assessment. Finally, we will discuss how these similarities and differences affected how readers from different backgrounds assessed the student papers.

Program Background

During each summer since 2007, the Department of Mathematics at the University of North Carolina, Asheville has hosted a Research Experience for Undergraduates (REU) sponsored by the National Science Foundation. During the eight weeks of this program, eight students from various colleges and universities worked alongside faculty in the university's Mathematics Department in seeking solutions to open research problems in mathematics. Most of these students had recently completed either their sophomore year or their junior year of college. (Only one of the twenty-four students taking part in the first three years of the program had just finished his freshman year.)

The REU program serves many explicit learning goals, including a few related to disciplinary writing. For instance, by the summer's end participating students are expected to demonstrate familiarity with the structure of a mathematical research paper and to be able to craft such a paper. Although a successful student need not complete a publishable research article by the conclusion of the program, the students spend the summer working on several iterations of a paper that may lead to such an article.

In order to help them meet the learning goals described above, student participants in the 2008 and 2009 REU programs were given instruction in all aspects of the math profession, including writing in the discipline of mathematics. In particular, students were introduced to LATEX, a powerful and flexible high-level programming language that allows students to produce technical documents featuring a nearly limitless variety of mathematical symbols and display environments (e.g., arrays, lists, and complex formulas). Throughout the program, the students regularly received a substantial amount of feedback on multiple drafts of their writing, both from the program's faculty mentors and from the other student participants. Feedback from faculty mentors came in the form of written comments on weekly drafts, and in one-on-one conferencing sessions. Near the end of the summer, students performed peer reviews of each others' work, guided by a rubric which helped to focus students' attention to specific aspects of one another's papers.

Assessment Methods

While the students in the REU produced eight drafts over the fourteen-week period, for this assessment we focused only on their final drafts.[2] In all, we examined 13 final versions, written by 14 students. (Our sample included one co-authored paper.) All four of us ranked the essays from high to low. While there was some disagreement about how to rank papers in the middle of the spectrum, we were surprised to discover how much we agreed on the top and bottom essays, which indicated to us that we were indeed operating with some of the same expectations for academic writing, even though only one of the researchers has mathematical expertise that allows him to evaluate the mathematical content. Our individual rankings of each student's text are presented in Table 1.

Table 1: Faculty Rankings of Student Texts

	Patrick	**Amy**	**Chris**	**Meg**
1.	Cory	Cory	Cory	Cory
2.	Lloyd	Stephen	Barbara/Dora	Stephen
3.	Bonnie	Lloyd	Beth	Lloyd
4.	Stephen	Barbara/Dora	Stephen	Beth
5.	Barbara/Dora	Beth	Frank	Barbara/Dora
6.	Mike	Betty	Bonnie	Frank
7.	Beth	Frank	Lloyd	Mike
8.	Morris	Bonnie	Betty	Bonnie
9.	Zelda	Rachel	Mike	Betty
10.	Frank	Mike	Zelda	Zelda

11.	Betty	Zelda	Morris	Morris
12.	Rachel	Morris	Rachel	Rachel

Our agreement is interesting to note because we responded to the students' writing as individual readers with unique and often highly contrasting backgrounds. Patrick is a professor of mathematics and the director of the REU program surveyed here. In addition, he directs his institution's writing-intensive program. Obviously, his experience as a math specialist factored into his assessment of each paper. The other three researchers, however, are not mathematicians, though they have varying expertise in composition, rhetoric, and writing across the disciplines. Amy is an English professor who specializes in rhetorical history and feminist rhetorics. She regularly teaches courses in first-year and interdisciplinary writing and has worked for WAC programs at two institutions. Chris researches composition pedagogy and the history of writing instruction and teaches first-year and interdisciplinary writing. He chaired his department's first-year writing committee. Meg is a published poet whose Ph.D. research addressed issues related to creative writing and composition pedagogy. She teaches courses in first-year writing, interdisciplinary writing, and creative writing.

Cross-Disciplinary Conventions

We identified five shared rhetorical conventions in the students' writing despite our disciplinary differences. These included the following:

- Transitions and metacommentary
- Style and tone
- Contextualization within the field of mathematics
- Use of sources
- Visual rhetoric

We now elaborate on our findings concerning each of these conventions.

Transitions and metacommentary

"Metacommentary" refers to a set of strategies writers use to announce their intentions. Comparing it to a Greek chorus, Graff and Birkenstein describe metacommentary as "as a sort of second text that stands alongside your main text and explains what it means. In the main text you say something; in the metatext you guide your readers in interpreting and processing what you've said" (p. 130). This metatext may come in the form of titles and subtitles that quickly preview the writer's argument, phrases such as "this paper argues" and "we intend to show" that pinpoint the text's most salient points, transitions that illustrate the relationship among a text's different claims, and paragraphs that explain how the remaining text is organized. Metatextual passages enable readers to efficiently unpack a text, identifying its purposes, its most important points, and its methodology. Inserting metacommentary into a text prompts writers, especially student writers, to reflect on the choices they make as writers and why these choices are effective. As Graff and Birkenstein argue, "When…students learn to use metacommentary…they get more out of their ideas and write longer, more substantial texts" (p. 132).

We identified examples of metacommentary in all the student papers we examined, and we observed that students frequently used metacommentary in their abstracts and introductions. In these sections, students tended to state in explicit terms their paper's central argument or purpose. For example, Stephen writes in his introduction, "The goal of this paper is to examine how 'expandable' a particular graph labeling is without changing the span of all labels." Beth states the purpose of her paper this way: "The question that interests us and motivates this paper is the expected diameter of a tree created by the Use It or Lose It process."

In addition, students wrote metatextual passages that connected different points made in the paper. This sometimes came in the form of sentences or paragraphs in which the writer previewed ideas examined more thoroughly in later sections of the paper. A particularly lengthy example of this strategy is found in Lloyd's article, whose introduction explains how the remainder of the paper is organized:

> In this paper, we describe a one-dimensional cellular automaton rule which generates variations of the Sierpinski triangle having significant overlaps (Figure 2). Specifically, we augment the original IFS by incorporating one or more mappings (with even spacing) along the bottom row of the Sierpinski triangle, and then describe an infinite-state CA which represents it. We then describe a method of computing fractal dimension of the objects based on the counting of non-zero cells at each successive stage of the CA. The necessary definitions and terminology are outlined in section 2, while the main results and several specific examples follow in Section 3. Proofs concerning correctness of the algorithm will be given in section 4, and direction for future work will be detailed in section 5.

Students more commonly connected their ideas by writing short phrases that referred back to earlier aspects of the article. In proving the "theorem that gives us the recurrent states for $m = n+b$, with a fixed b," Zelda, for example, references theorems and lemmas introduced earlier in the paper: "Our last statement is true *due to Theorem 1.1*. Finally, *by lemma 2.1 again*, ..." (emphasis ours).

One of the most sophisticated metacommentary strategies we observed, which not all students used, was when the writer anticipated and addressed potential questions readers might have. Dora and Barbara make such a move in the following passage: "[W]e chose to examine how graphs grow by preferentially adding edges between pairs of adjacent vertices or pairs of vertices connected by some path of length 2. For simplicity, we do not consider pairs of vertices connected by paths of length greater than 2, but we also foresee generalizations of our work in that direction." In the second sentence, Dora and Barbara recognize that some readers may wonder why the paper only examines certain pairs of vertices; they explain their rationale ("For simplicity") and speculate that their conclusions may very well apply in other cases.

Effective uses of metacommentary like the ones illustrated were important in our assessment of students' articles, especially for members of the research team not trained as mathematicians. When used effectively, metacommentary gave them the impression that the student possessed a solid understanding of her project and its significance. When used poorly, metacommentary signaled to these readers that the writer did not fully understand basic conventions of mathematical writing and academic writing in general. For example, Betty uses metacommentary throughout her paper, but in doing so, she makes explicit comments professional scholars would leave unstated, which caused readers to doubt her authority. Before defining her key terms, Betty writes, "Basic definitions and theorems are the easiest place to start, which is exactly where we will begin. The following are key definitions and notations used throughout the rest of this paper." Betty's first sentence is unnecessary both logically and conventionally. In other words, readers don't need it to understand the following sentence; furthermore, it's commonly understood among members of the academic mathematics community that an article will only begin with a definition if that term is needed to say something intelligent about the matter at hand, in which case the definition will be introduced without much fanfare, as a matter of course. Most importantly, though, Betty's first sentence violates tacitly understood conventions of academic decorum. Academic writers typically start with a complex idea or question, not with something "easy" as Betty does here.

Style and tone

When considering the elements of effective writing, issues of style and tone prove to be some of the most difficult discussions to have, not only with students, but also with colleagues. What, specifically, makes a sentence awkward or vague? How do we describe to a student or to each other what kind of tone we're looking for? Even more importantly, does what we're looking for, or perhaps listening for, vary between disciplines? Often when responding to student writing, the language we use to discuss style is vague and tends to rely on metaphors of listening or feeling. Our early assessment discussions mirrored some of the difficulty in defining what it was that "sounded wrong." Most often, we don't consider the nuts and bolts of sentence construction upon first reading. We get a "feeling" for how sophisticated a piece of prose is and we might even describe tone in vague terms like how a writer "comes across."

A closer examination of sophisticated academic writing often reveals relatively complex and self-consciously patterned sentences that display balance and consistently professional vocabulary. Less sophisticated examples are often labeled either wordy or cluttered, which is sometimes the result of a string of prepositional phrases, lots of articles, or heavily modified verbs. In the opposite camp, student writing can also be labeled thin, superficial, vague, or redundant, which may be caused by repeated words that are too close together or by a lack of variety in sentence pattern and length. A comparison of the abstracts and/or introductory paragraphs submitted by four students provides a compelling example of the rhetorical importance of an author's style and tone considerations. For any reader without specific knowledge and experience in mathematical writing, these samples may also provide a window into the process of attempting to make meaning in a sea of mathematical jargon.

In the first example, Rachel titles her paper "The Search for an Upper Bound on the Number of Graceful Labelings of a Path with *N* Edges," and writes the following abstract:

> The concern of this paper is to provide an effective way to measure the rate of growth for the number of graceful labelings of a path graph with *n* edges, as *n* increases. We introduce the *graceful labeling diagram*, which we use to systematically construct graceful labelings, and develop analytical tools that exploit the structure of the diagram to compute an upper bound on the number of graceful labelings of a path.

The compositionists on the research team first noticed the cluttered nature of the title, as well as the first and second sentences, which are built through a series of prepositional phrases, a sentence strategy Rachel uses throughout the paper. Patrick concurred: this construction is not standard in mathematics any more than it is in any other discipline. We read her over-reliance on this single sentence-building style as lacking syntactical variety and complexity. Additionally, Rachel uses the term "graceful labeling" three times in her second sentence. When studying the introductory paragraph in combination with her abstract, the term appears twelve times. While repetition can be an effective rhetorical strategy, this particular pattern sets up several circular sentences where the subject being defined is also a part of its own definition. Perhaps this repetition was more noticeable to the non-mathematical expert readers precisely because we were trying to derive the meaning of a "graceful label" from the context. In addition to Rachel's repetition of key terms and overuse of prepositional phrases, her prose also lacks syntactical sentence complexity, as seen in the following example from her introduction: "Alexander Rosa introduced the concept of a graceful labeling in his 1967 paper. He defined a graceful label of a graph as ' ' [sic]. Since then, extensive research has been done regarding graceful labelings." The succession of these three short sentences creates a feeling of emptiness–ironically echoed in the empty quote that's included in her final draft—suggesting that she only has a superficial understanding of the term and concept being presented. One of us commented that sentences like these reflected a "lack of depth" and seemed "book- reportish," and we also noted the stylistic weakness of the third sentence, which simply resolves the significance of the quote by suggesting that "extensive research has been done." Finally, these three sentences all use a simple sentence construction, which stylistically results in short, choppy, and seemingly disconnected sentences, which, as one of us noted, could have easily been combined using a compound-complex structure.

For a contrasting example of sentence variety and complexity, we may again examine Lloyd's abstract:

> It is well known that Pascal's triangle can be generated with a cellular automaton (CA) construction. Moreover, taking cell values mod 2 yields an approximation of the Sierpinski triangle. It is then natural to ask: What other fractal objects may be represented by cellular automata? Specifically, given a variation of the Sierpinski triangle's iterated function system which includes significant overlaps, can we identify a CA which describes it? We will describe a specific family of fractals which are related to the Sierpinski triangle and which include overlap, and then give a representative cellular automaton construction, with an inductive proof of equivalence.

In addition to using more complex sentences, Lloyd also engages the reader and adds stylistic variety by posing two questions in his abstract. We described his tone throughout the paper as confident and clear. We also noted, in contrast to a paper like Rachel's, that Lloyd's diction choices were increasingly more specific, instead of repetitive. Often Lloyd includes parentheticals to further clarify or offer additional information. Other times he states overtly that he is going to describe something, "specifically," or "more specifically," as in the following example from his introductory paragraph: "In this paper, we describe a one-dimensional cellular automaton rule which generates variations of the Sierpinski triangle having significant overlaps (Figure 2). Specifically, we augment the original IFS by incorporating one or more mappings (with even spacing) along the bottom row of the Sierpinski triangle, and then describe an infinite-state CA which represents it." Stylistic choices like these help Lloyd develop a consistent and convincing tone that persuaded non-expert readers to trust the math behind the writing.

The abstract submitted by Betty mirrors some of the flaws found in Rachel's earlier example. Like Rachel, she chooses repetitive language and her sentence structure lacks complexity:

> This talk [sic] will address necessary properties for a graph to be W*n,m*-color-critical. Some specific examples will be analyzed. In particular, we will show that there does not exist a connected graph with seven vertices that is W*4,2*-color-critical. Also, a general color critical relationship between complete graphs and wheel graphs will be discussed. These results answer questions raised by Nesetril and Nigussie.

Here she relies on the same subject-verb-direct object progression, using verb choices such as "will address," "will show," and "will be discussed." In her introductory paragraph, she continues this pattern with "will begin" and "will then prove." One interesting variation in Betty's prose style appears intermittently throughout her paper. In several places, Betty gestures toward her audience by trying to include us in what one of us described as a casual conversation. Sentences like the following create a much more informal tone: "Now that our basic definitions are clear, let's look at…" or "the only other option here would be to…" or "Interestingly enough…the idea of cones and also suspensions could shed some light on finding color-critical graphs, so let's look at…" While the non-mathematical expert readers appreciated the friendliness of this approach, the shift in tone between these kinds of sentences and other more formal, math-based sentences was jarring. Ultimately, we suspected that these kinds of tonal shifts indicated attempts to mask some of the gaps in this student's understanding of her larger project.

Finally, Stephen's paper stood out as a particularly smooth and "fluid" example of style and tone. Consider the following passage taken from his introductory paragraph:

> A considerable amount of research has been done on $L(2, 1)$-labelings and the more general $L(h, k)$-labelings. The origin of this category of problems is channel assign- ment where broadcast channels for various nodes are assigned such that there is no interference with each other, while minimizing the frequency spectrum used. For a survey of the problem, see [2] as well as [3] and [4]. The goal of this paper is to examine how "expandable" a particular graph labeling is without changing the span of all labels. We call this the *utility* of a labeling.

Here we note the effectiveness of Stephen's use of italicization and quotation marks to draw attention to key concepts he is working to define. It's as if he is calling his audience's attention to the fact that he is entering these terms into the larger conversation. This technique is much more rhetorically effective than simply stating, "Let utility be defined as…" like many authors did in similar circumstances.

A quick glance at Stephen's sentence variety as well as his ability to construct complex sentences also persuaded us that he is in control of his project. Like Lloyd's earlier example, Stephen's prose style complements the complexity of his larger project. Both writers, as well as several others not examined specifically in this section, create variation in diction, sentence pattern, and length, often combining choppy sentences or moving adverbial clauses or participial phrases to the beginning of their sentences. The ease and fluency with which Lloyd and Stephen describe their work led us to believe that their overall grasp of the project was more expansive as well as more specific than others.[3]

Contextualization within the field of mathematics

As in any discipline, one marker of effective writing is the extent to which students situate their work in relation to ongoing conversations in the field. Effective writers had a tendency to use several techniques to situate their work including giving a history or background of the problem they were studying, making connections to other articles on the field, making explicit statements about the potential practical applications of their work or the significance of their work to the field, and the ability to project future work that might be taken up in light of their conclusions. By performing these moves, writers demonstrate an awareness that academic work doesn't get done in a vacuum, an understanding that was lacking in less effective essays.

Perhaps the most developed examples of contextualization appeared in Dora and Barbara's collaboratively written article. Take, for example, the section of their paper they title "Context." It is significant that they not only provide the context for their work, but also explicitly label it as such. In this section, they situate their research as an "amalgamation of previous studies." They first point out that "while much work has been done on detecting communities within pre- existing networks, our work assumes that the vertices are communities themselves." In addition to questioning the assumptions of prior research, Dora and Barbara consider what areas of study their work might apply to: "Other work has investigated preferential attachment in the growth of communty structure….[O]ne can apply our growth model to this area of study." At the end of this section, they distinguish their project from existing studies on networks: "Another focus of previous work is geographical attachment in random graph generation. […] Our growth model similarly generates random graphs without preferential attachment but a somewhat different notion of geographical attachment." Even without an understanding of the mathematical problem they're working on, readers can see that Dora and Barbara understand how their work connects to existing research while differing in important ways.

As mentioned in the metacommentary section above, in their introduction, Dora and Barbara also gesture generally towards further work they might do in light of their current results, which they then return to more specifically in the conclusion, mapping out three specific directions their work might take: expansion of their current study, modification of the models they worked with, and "find[ing] and test[ing] applications of the model." Not only are they talking about further work, they emphasize that their work might have real-world application to networks and that their project applies to questions outside of math.

Weaker essays made unsuccessful attempts to contextualize their work in the field, or in Morris's case, very little attempt at all. Here, for example, is his opening paragraph:

> For certain path-like constructions, we show that by appending a cone of a graph at each vertex, this new graph will have a symmetric unimodal independence polynomial. We also consider a few examples for which this process will work, but are not path-like. We provide here some definitions fundamental to this paper.

While this passage does display a sense of purposefulness, it lacks any explanation of the background of this problem, its significance in the field, or its connections to other scholars' work (explanations which are also are not offered in other sections of the essay). Furthermore, unlike some of the more successful writers, he also doesn't include a final section that would contextualize his work in terms of what his results mean:

> We define a general *Kt* path to be a path of *Kt* graphs, each connected to the previous along a *Kt−1* graph. Applying Proposition 2.2 we obtain this formula for the independence polynomial of a general *Kt* path made of *n* many *Kt* graphs:
>
> pn(x) = b(x)[pn−1(x) + x · c(x)b(x)t−2pn−t(x)]
>
> where *b(x)* and *c(x)* are defined above. If the same conditions hold for *b(x)* and *c(x)*, then this should be a SU. independence polynomial.

His final paragraph assumes that readers will know the significance of this final statement. Perhaps a seasoned mathematician might be able to extrapolate some significance, but Morris fails to understand that his audience expects him to state the significance of his argument and situate it in relation to other research in the field. His inability to contextualize within the field, in combination with his over-reliance on jargon, might be read as a marker of his amateur status. As Charney and Carlson (1995) note in their research on students learning how to write up psychology experiments, "novice writers, novice researchers, and novices in the discipline" have difficulty "anticipating what readers will find interesting or controversial" because "they have little of the tacit experiential knowledge that full-fledged scientists rely on" (p. 89). Unlike Dora and Barbara, who appear to understand the relationship between their project and others in the field, Morris offers us no way to understand the significance of his project to the larger field.

Use of sources

Just as would more experienced writers, the students we surveyed made a variety of uses of the sources they found. The students who wrote most effectively were able to make sophisticated use of their sources in order to meet the need to contextualize their work; the need to draw on others' earlier work in order to build upon it, amend it, or emend it; and the need to find yet other sources.

We identified the following uses of sources in the students' work. This list is given in order from most frequent usage to least frequent.

1. To draw on a result from the source in order to support one of the student's propositions or claims
2. To contextualize the student's work in the body of earlier work
3. To extend results appearing in the source or to point out a problem in the source and fix or resolve it
4. To find other sources

As indicated above, we may think of each of these purposes as meeting a particular need. For us, then, a student's use of sources to meet a particular need was considered effective if that need was met by the sources the student found and referred to. We will soon see that the students used sources most effectively to meet the first need indicated above; the other needs were not so well met, except by a few of the most talented writers.

Recall that we received final drafts of 13 student papers. All of the data given below were drawn from these papers. Most students cited few sources, substantially fewer than more experienced mathematical writers would. However, greater quantity did not always result in greater quality: the stronger writers often referenced as few sources as their less able peers, and indeed the authors of one of the most rhetorically sophisticated articles eliminated sources from their bibliography as their work evolved.

There were a total of 45 distinct sources cited by students and 80 individual in-text citations, so that each source was cited roughly 1.8 times on average. (Here we may count a source multiple times, according to the number of students who cite it.) It is worth noting that one student (Cory) accounted for 20 (or 25%) of 80 individual citations. Of the 80 citations, 28 of them (35%) served more than one of the uses given above simultaneously. Cory was responsible for 10 (roughly 35.7%) of these simultaneous uses.

Table 2 below summarizes the uses served by the individual citations.

Table 2: Uses Served by Individual Citations

Use	Frequency	Percentage
To support student's claims	46	57.5%

To contextualize student's work	41	51.3%
To extend or emend an earlier author's work	20	25%
To find other sources	3	3.8%

From the data above we can immediately infer that students made effective use of their sources for the task of obtaining a fact or result which would later be used in the course of their own work. (This task is a particularly important one in content-driven fields like mathematics.) On the surface it appears that nearly as frequently students used sources to contextualize their work in relation to other scholars in the field. Indeed, the stronger students' use of sources for this purpose compares favorably with that of the professional mathematicians surveyed by Burton and Morgan (2000): Cory, Mike, and Stephen each made at least 5 citations with the aim of contextualizing their work within the existing body of knowledge, more such citations than at least 33 of the 52 (63.5%) of the mathematicians surveyed by Burton and Morgan (2000).

However, once these three highly effective writers were factored out, the remaining student papers contained merely 2.2 citations, on average, of this second variety. This small of a number of citations is typically insufficient to provide proper context for a scholar's work, and thus overall the students' effectiveness in meeting the second need is not as great as that of more experienced mathematical writers. The students were similarly ineffective at meeting the third and fourth needs, which effectiveness becomes especially evident once we note that a single student (Cory) accounted for over half (55%) of the citations used to refer to prior work in order to extend or generalize it. Perhaps most significantly, only three students used a source to find other sources. Not surprisingly, these were the same three students (Cory, Mike, and Stephen) who met this fourth need most effectively.

Finally, we may gain some understanding of the students' effectiveness in using sources by examining the types of sources they used. Of the 45 sources used, 36 (80%) were journal articles, 6 (13.3%) were books, 2 (4.4%) were websites, and 1 (2.2%) was a set of unpublished notes produced by one of the faculty members participating in the program. In mathematics, as in many other content-focused disciplines, cutting-edge research takes place primarily in scholarly journal articles, and therefore the prevalence of such articles in students' lists of sources is appropriate and demonstrates that the students were able to seek and find such sources (with or without faculty help) effectively.

Visual rhetoric

Professional mathematical writing contains a large amount of notation, much of which is often grouped into "displayed formulas," blocks of centered, symbol-laden text which is separated from the surrounding paragraphs by a small amount of whitespace. In-line formulas are also common.

The overuse of notation results in an excessive amount of displayed formulas and short, notation-heavy sentences that have a jarring visual effect on the reader, especially when these formulas are inexpertly paginated. The effect is more than visual: notation-heavy mathematical writing emphasizes formulas at the expense of expository prose, resulting in lower readability. As one of us noted, the strongest students' writing appeared "chunkier" on the page, and the more substantial expository paragraphs allowed the reader to move more quickly through the paper, lending a greater sense of understanding.

Other visual markers also give the writer the appearance of professionalism (by following various print conventions used by published journals, for example) and, at the same time, they give readers cues that make the text easier to navigate. Four visual qualities affected our evaluations of the students' essays: (1) the use of visual organizing devices to create sections and subsections, (2) the integration of images into the text, (3) use of white space and the use of alignment tabs to create meaningful and visually pleasing layout choices, and (4) the use of mathematical notation in displayed versus in-line formulas.

1. Use of visual organizers

One important visual device is the use of visual organizers such as use of headings (for sections and subsections), indentation, bullets, and numbered lists. Visual organizers help readers understand relationships between various portions of the text and show that the writer understands how to break the argument down into smaller, easier-to-digest chunks, which increases readability. The effective use of these devices requires that students know what organizing conventions are typically used in math publications, and they also have to know what formatting choices are possible in LATEX.

Stronger writers seemed to display very conscious and deliberate choices, utilizing a variety of devices. For example, Dora and Barbara used a combination of organizers to break their argument into smaller pieces and to create hierarchical relationships between them. For example:

> 2. Definitions and Examples
>
> 2.1. **Growth model.** This section delineates the mechanics for growth of the multigraph model, defined as follows:
>
> (1) *Initialization*
> Start with any connected multigraph without loops but permitting multiple edges that has an ordered labeling of the vertices. Fix α, β, and $\gamma \in \mathbb{R}^+$.
>
> (2) *Growth*
> At each time step, exactly one new edge is added between existing vertices. Also possible is adding a new vertex and merging two vertices. Let the distance between two vertices $u$and v, $d(u, v)$, be the length, n, of the shortest path between u and v.

Adding edges:

> At each time step, examine each pair $\{u, v\}$
>
> Let $|Eu,v|$ be the number of edges (counting multiple edges) between u and v, and let k be any vertex forming a path of length 2 from u to v.

Within this short span, Dora and Barbara use a combination of devices–including centering, numbering, bolding, bulleting, spacing, and indenting–to pull apart the various pieces of their argument and to help the reader locate their place in the hierarchy of their points and subpoints. Weaker writers did attempt to use some organizers, but had a tendency to use fewer of them, with less variety, as well as fewer signals to readers about the different parts of their argument and their relationship to one another.

2. Integration of images into the text

Another important aspect of visual rhetoric that affected our evaluation of the student papers was the extent to which students effectively integrated various images into their written text. Such explanations are important since they help readers find the appropriate figure in the text (which may or may not be printed near the section discussing it depending on the publisher's particular printing choices). More importantly, well-developed explanations help readers more clearly understand the purpose of each image in relation to the argument being made by the text. For example, Stephen's integration of his third figure demonstrates that he understands the convention of referring to images, and that he also understands that the argument made by an image needs to be explained and connected to the text. (See the excerpt below.)

Syntactically, Stephen has pulled the infinitive verbal phrase "To see how…" to the beginning of the sentence, which effectively introduces the figure by highlighting its purpose for the reader. He further explains the image by emphasizing what the reader is supposed to take from the figure with phrases such as "In Figure 3, we see that…." He also helps readers focus on different parts of the image and their relationships to one another through phrases such as "However, in the first case…" which focuses readers on the left half of the image, which is then related to the right half of the image with the phrase "while in the latter…." Throughout the passage, he makes lots of connections for the reader between what he is saying in the passage and how it relates to the figure he is showing, including printing the formula for each half of the image inside each image and then making a connection by repeating those formulas again in the text.

To see how different labelings realizing the span of a graph can yield different utility, we look at the prism Pr_{14} in Figure 3.

From Proposition 2.4 the labeling blocks A = $\begin{matrix} 1 & 5 & 3 \\ 4 & 0 & 6 \end{matrix}$, B = $\begin{matrix} 1 & 5 & 0 & 3 & 6 & 0 & 3 \\ 4 & 2 & 6 & 1 & 4 & 2 & 6 \end{matrix}$, and C = $\begin{matrix} 0 & 5 & 2 & 4 & 6 & 1 & 5 & 3 \\ 4 & 1 & 6 & 0 & 2 & 4 & 0 & 6 \end{matrix}$. In Figure 3, we see that l_1 is built by *AAC*

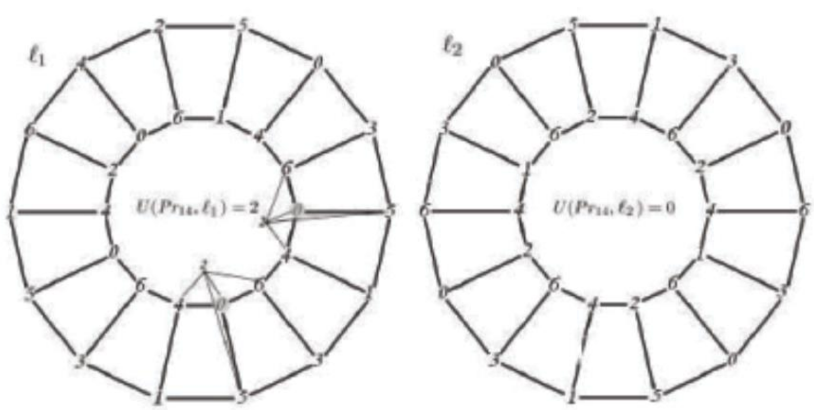
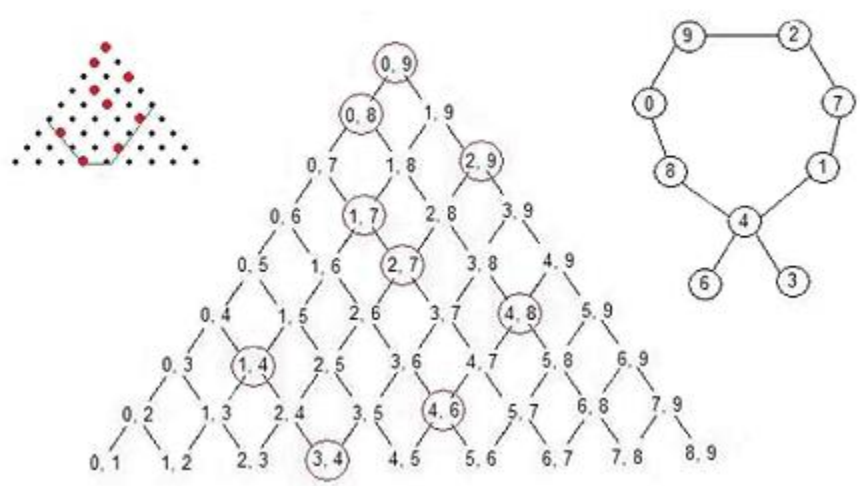


Figure 3. $U(Pr_{14},l_1)$ and $U(Pr_{14},l_2)$

and l_2 is built using *BB*. However, in the first case $U(Pr_{14},l_1) = 2$ while in the latter $U(Pr_{14},l_2) = 0$.

Less effective writers simply gave readers more work to do to figure out what the figures were meant to illustrate. Take, for example, Rachel's integration of figures. To introduce her first figure, she writes, "We introduce the diagram in Figure 1 for generating gracefully labeled graphs." Compare Rachel's integration to Stephen's introduction of Figure 1 that syntactically emphasized what purpose the figure served. Rachel puts the purpose of her image at the end of the sentence, choosing instead to emphasize the construction "we introduce," which highlights instead a kind of unnecessary metacommentary (i.e., if she had introduced the figure well, she wouldn't need to announce that she is introducing it). Rachel also tended to assume that her figures would speak for themselves, which resulted in vague introductions such as "See Figure 2 for an example." Like Stephen, Rachel produced multi-part figures, but she doesn't help the readers explicitly navigate the different parts of the image, as seen in her use of her Figure 3.

> **Definition 0.5.** We say a position is a ***candidate*** if it is able to be selected.
>
> The first rule ensures that the labeling is graceful; the second ensures a limit to the degree of the vertices. No graceful graph created using these rules will contain a vertex with degree greater than 4; further, most vertices will have degree no greater than 2 (See Figure 3 for an example). Graphs created by following these rules are called *pseudopaths*. This is so because although all paths are included in the set



> Figure 3. The configuration above is valid, however, the corresponding graph has a vertex with degree 4 of configurations valid by these rules, cycles and other graphs resembling paths are also included.

Absent from this excerpt is any language that directs the reader's attention to different pieces of the image. Of course, an experienced mathematician, knowledgeable about the objects Rachel refers to, will be able to figure out her point, but Stephen's integration is likely to read as more sophisticated and experienced prose.

3. The visual effect of whitespace

Effective use of white space is essential in producing a rhetorically effective piece of mathematical writing. When used effectively, white space goes unnoticed; when used ineffectively, its clumsy use becomes jarring. Beginning mathematical writers, those less skilled in the use of the built-in LATEX commands which often make visually appealing alignment, justification, and vertical spacing automatic, are more apt than seasoned mathematicians to craft documents containing such unsettling white space. LATEX comes equipped with a number of "environments," which are essentially macros with preset formats concerning alignment, justification, typeface, etc. That is, an environment offers students a template whose use helps students adhere to standard mathematical formatting conventions.

For instance, LATEX's array environment makes it easy for the writer to create tables, arrays, matrices, and other objects whose effective rendering requires both vertical and horizontal alignment. This environment would have helped Beth in her attempt to paginate the complicated expressions appearing in the final draft of her paper. First we reproduce a block of text as she rendered it:

$$\tau(\{(2,d,0),(2,d+1,0),(1,d+2,0)\},(2,d,1));\tau(\{(2,d,0),(2,d+1,0),(1,d,0)\},(2,d+1,1));$$

$$\tau(\{(2,d,1),(2,d+1,1),(1,d,1),(1,d+1,1)\},(1,d,1));$$

$$\tau(\{(2,d,1),(2,d+1,1),(1,d,1),(1,d+1,1)\},(1,d+1,1)).$$

To create the above text Beth used three distinct instances of a "displayed equation" environment to render mathematical formulas which are centered and set off from the main body of the text in order to emphasize these formulas or to make them more clear (in much the same way that block quotations are used in writing in other disciplines). Because this environment automatically centers all text appearing inside of it, each line is individually centered. Consider the appearance of the same text when two changes are made to the LATEX code: (1) a single displayed equation is used, and the text is placed into a simple instance of the array environment, which allows left justification, and (2) variable sizes of "delimiter" symbols (parentheses and braces) are used to enhance the order of their appearance in the expression:

$$\begin{array}{l}
\tau\Big(\Big\{(2,d,0),(2,d+1,0),(1,d+2,0)\Big\},(2,d,1)\Big);\\
\tau\Big(\Big\{(2,d,0),(2,d+1,0),(1,d,0)\Big\},(2,d+1,1)\Big);\\
\tau\Big(\Big\{(2,d,1),(2,d+1,1),(1,d,1),(1,d+1,1)\Big\},(1,d,1)\Big);\\
\tau\Big(\Big\{(2,d,1),(2,d+1,1),(1,d,1),(1,d+1,1)\Big\},(1,d+1,1)\Big).
\end{array}$$

The array environment is a tricky tool to master, and students often misuse it in their initial attempts at producing visually effective mathematical documents. Consider Zelda's attempt to use the array environment to render a series of logical implications:

$$\begin{array}{rcl}
(x_1+1)+\ldots+(x_{n-1}+1) & \geq & (n-1+b)(n-1)+r\\
\Rightarrow (x_1+\ldots+x_{n-1})+(n-1) & \geq & (n-1+b)(n-1)+r\\
\Rightarrow (x_1+\ldots+x_{n-1}) & \geq & (n-1+b)(n-1)+r-(n-1)\\
\Rightarrow (x_1+\ldots+x_{n-1}) & \geq & (n-1)((n-1+b)-1)+r\\
\Rightarrow ((n-1+b)-r,x_1,\ldots,x_{n-1})\in R(n-1+b,n). & &
\end{array}$$

Zelda has used three columns, right-justifying the first column, centering the second, and left-justifying the third. The effect is to highlight the role of the inequality, symbolized by "≥" in each line. However, the most important symbol for Zelda's actual argument is the "implies" symbol, "⇒," whose function is obscured, since every "⇒" rides the ragged left edge of the expression.

In comparison, in crafting the following series of equalities, Lloyd recognizes that the symbol "=" is the highest-order relational symbol, and its central role is emphasized by his layout choices:

$$\begin{aligned} g_{2^{k+1}-2,2m} &= g_{2^{k+1}-2-2^k,2m-2^k} + g_{2^{k+1}-2-2^k,2m} + g_{2^{k+1}-2-2^k,2m+2^k} \\ &= g_{2^k-2,2m-2^k} + g_{2^k-2,2m} + g_{2^k-2,2m+2^k} \\ &= g_{2^{k-1}-1,m-2^{k-1}} + g_{2^{k-1}-1,m} + g_{2^{k-1}-1,m+2^{k-1}}. \end{aligned}$$

Note that Lloyd aligns columns in exactly the same manner as Zelda; however, the difference lies in the choice of the symbol being emphasized. Because of the length of the inequalities appearing in Zelda's expression, it is impossible to render the expression with the same alignment restrictions without causing the text to bleed into the margin and produce a different sort of visual unpleasantness. However, by adopting a two-column approach we obtain the following more effective rendering:

$$\begin{aligned} & (x_1+1)+\cdots+(x_{n-1}+1) \geq (n-1+b)(n-1)+r \\ \Rightarrow\ & (x_1+\cdots+x_{n-1})+(n-1) \geq (n-1+b)(n-1)+r \\ \Rightarrow\ & x_1+\cdots+x_{n-1} \geq (n-1+b)(n-1)+r-(n-1) \\ \Rightarrow\ & x_1+\cdots+x_{n-1} \geq (n-1)((n-1+b)-1)+r \\ \Rightarrow\ & \big((n-1+b)-r, x_1, \ldots, x_{n-1}\big) \in R(n-1+b, n). \end{aligned}$$

We note that in the sums appearing in this last expression we have also made the very minor changes of removing extraneous parentheses and replacing the ordinary ellipses, "...," typically used to denote omission from a simple list, with the centered ellipses, "· · · ," denoting here omission from the arithmetical operation of summation. These minor adjustments, though subtle, have a profound impact on the persuasiveness of the writing.

4. The visual effect of mathematical notation

Finally, the decision to use or not use various elements of mathematical notation can also affect the visual effectiveness of a piece of writing. Although mathematical symbols and other notation provide convenient shorthand for more lengthy terminology, the use of these symbols is not unrestricted. For instance, various symbols with relatively short English equivalents (e.g., the symbol "⇒" used above for "implies," "∃" for "there exists," and "∀" for "for all") are stylistically appropriate in some contexts and not in others.

Whether or not a symbol is permissible may depend on the textual context in which it appears. For instance, the implicational symbol "⇒" is most commonly (and appropriately) used in a statement rendered in other mathematical notation, as Zelda has above. Though the symbol's use in a statement rendered in plain English is not technically incorrect, its use in such a context is visually startling and stylistically weak. Consider the different effects produced in the reader by the following statements, formally identical given the obvious meaning of the other notation appearing, in which the symbol "⇒" is employed: "The degree of the vertex is 1 ⇒ it is a leaf of the tree T" and "$d(v) = 1 \Rightarrow v \in L(T)$." Again, though the first is technically correct, it is stylistically awkward. The most effective student writers tended to be aware of the importance of textual context in deciding whether to use a piece of notation or not.

Bridging the Disciplinary Divide

So far we have explored the common ground shared by mathematicians and composition and rhetoric specialists. From the perspectives of both mathematics and composition teachers on the research team, effective student writing in our sample performs the following conventions:

- Transitions and metacommentary emphasize and connect the paper's most important points
- Stylistic choices display variety and complexity
- Sources were used for a variety of purposes
- Format and layout choices demonstrate familiarity with disciplinary conventions and enhance the writer's argument

In the remainder of this section, though, we want to briefly address what students' papers taught us about the differences between academic writing in mathematics and in the humanities. Some of the differences we observed in our individual evaluations of students' writing could be ascribed to these disciplinary differences, but they also arose from our different sub-specializations and our individual proclivities as readers. As Thaiss and Zawacki (2006) would explain it, at play in our readings were not only "disciplinary preferences but also…subdisciplinary and idiosyncratic preferences" which all play a role "any time a teacher evaluates student writing" (p. 61).

First-person plural is the convention in mathematical writing, whereas in the humanities writers are unlikely to use the first-person plural though they may use the first-person singular depending on the audience and the field. However, it's never wrong in the humanities to simply default to third-person. We furthermore noticed different conventions concerning the use of imperatives. While writers in the humanities may occasionally use imperatives, writers in mathematics frequently use imperatives to lead readers through proofs, to establish assumptions and common knowledge, and to define key terms.

These differences were easily overlooked by those of us in the humanities once we understood they were conventions. There were, however, other disciplinary differences that were harder to read past when evaluating students' work. For example, academic writers in mathematics can reference sources with minimal contextualization and explanation. It's acceptable for a writer, as Morris did, to simply state, "The propositions we use, which come from [1], are as follows." While Morris clearly links to his source, if he were writing his paper for a humanities class, his teacher would likely ask him to make more explicit connections to the text by employing signal phrases, naming the authors and sometimes titles, and by summarizing the work or contextualizing that part of the text from which the quote or paraphrase is taken.

Likewise, it wasn't unusual for the compositionists on the research team to rank lower those papers whose concluding sections were either brief or absent. Without these formal conclusions, they were unable to see the stakes of the students' research or where this research was heading next. Patrick, our disciplinary expert in math, stated that fleshed-out conclusions in mathematical writing are more of a stylistic option than a rhetorical necessity.[4] There are also several styles of writing in math, one more narrative and one less narrative, and Patrick noted that both were equally acceptable, though he prefers the narrative style himself. The less narrative style is more likely to represent more of its thinking through mathematical notation than prose. It is not surprising that those of us in the humanities preferred a more narrative style, as it tends to use more language that is easier for non-experts to understand. It is important to note, though, as Patrick pointed out, that the less narrative style could still get published (depending on the publication, of course) if the article explores a very significant problem.[5]

Of course, the extent to which some of us could actually understand the math also affected our assessment of the students' projects. While the top papers did tend to display both mathematical and rhetorical prowess, there were mathematically sophisticated papers that lacked the same polish as some of the other top essays. One essay in particular where this affected our rankings was Bonnie's essay which Patrick had ranked third, because, as he told us, the project explored a very significant and publishable problem. However, without that knowledge, the rest of us ranked her much lower because her rhetorical choices seemed less sophisticated than the other top essays. Likewise, Betty's essay, which Patrick noted explored a less significant mathematical problem, and which he ranked second to last, was ranked higher by Amy, Chris, and Meg, who believed that Betty's rhetorical choices–especially her decision to stake out further questions–mitigated some of the paper's problems.

We recognize that the differences we note here skim the surface and that more work needs to be done on how students learn disciplinary writing in mathematics, as well as what pedagogical approaches best support students' development as writers and participants in the mathematics discourse community. Our findings were limited by the fact that we focused our attention on students' final drafts; future studies might more fully investigate students' writing processes by examining students' reading, research, and revision practices. However, what we have tried to stake out here is the "tremendous implicit agreement" that Graff (2000) claims writers across disciplines share. In fact, we were surprised at how many assumptions about good writing we had in common, even though some of us were worried that mathematical writing would be completely alien. We hope our research can help build bridges between first-year and disciplinary writing. Better understanding of disciplinary convergences and divergences can help students make more informed rhetorical choices and help faculty design assignments and activities that prompt students to engage in this work.

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Notes

[1] Recent textbooks such as *Reading and Writing in the Academic Community* (4th ed.) and *Academic Research and Writing* publish no readings that focus explicitly on mathematics. The only example of writing from math included in *Issues: Readings in Academic Disciplines* is a report, originally published as part of the 2001 *Proceedings of the National Forum on Quantitative Literacy*, on the importance of statistical thinking to quantitative literacy initiatives (79-98).

[2] Our future examination of students' development as mathematical writers will look more closely at students' writing processes by examining both their drafts and responses to a program survey.

[3] In the above section both of the samples identified as stylistically complex were written by male students while the two less effective samples were authored by females. This in no way indicates that all the writing produced by female students in this study or in the broader field of mathematical writing is less complex. Our population sample was small, and several of the most successful female student writers are discussed in greater length in other sections.

[4] We realize that earlier in the essay we criticize Morris' paper for not including a conclusion that states the significance of his argument. Our issue with Morris' paper is less about the lack of a conclusion per se and more about his inability to contextualize his work within the field. Professional mathematicians who forego a concluding paragraph will still state the significance of their findings elsewhere in their article.

[5] Although there are these range of styles, some students' writing contained so little narrative that their work would not yet be publishable in a mathematics journal.

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